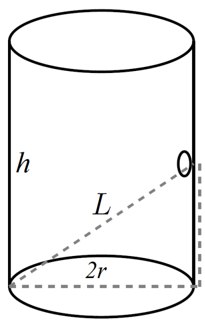
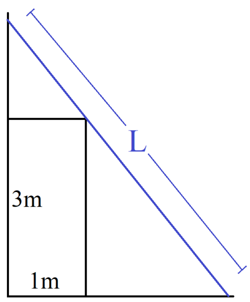
MATH 102:107, CLASS 16 (FRI OCT 13)

(1) (Constrained optimization - Kepler's wedding) A cylindrical wine barrel has a hole in the center of one side. When a rod is put into this hole and reaches the furthest into the barrel that it can go, it reaches a distance of L. Given this constraint, find the radius r and height h which maximize the volume of the barrel.



(2) (Constrained optimization) A box of height 1m and depth 3m is placed against a wall. A straight ladder must go over the box and lean against the wall. What is the shortest possible length of the ladder?



(3) (Constrained optimization) *Baculovirus* is a cylindrically-shaped cell which must hold a certain amount of genetic material, and therefore has fixed volume 54000π nm^3. Find the radius and height which give the cell the minimal possible surface area.

(4) (Unconstrained optimization) Let x measure the population of aphids in a garden. The reproduction rate of aphids is $G(x) = 3x$ and the rate of predation by ladybugs is $P(x) = \frac{30x}{5+x}$. Is there a value of $x > 0$ for which the net growth rate is *minimized?* At which it is *maximized?* For each, either find the value of x, or explain why none exists.

(5) (General problem solving) Let $f(x) = 4 - x^2$. Calculate the equation of the line(s) passing through $(5/2, 0)$ tangent to the graph of $y = f(x)$.